# roots of quadratics

1.

(i) Write down the condition for the equation $ax^2 + bx + c = 0$ to have no real roots.

1

(ii) Hence or otherwise show that the equation $x(x + 1) = 3x - 2$ has no real roots.

2

2. Show that the roots of the equation $(k - 2)x^2 - (3k - 2)x + 2k = 0$ are real.

4

3. Given that k is a real number, show that the roots of the equation $kx^2 + 3x + 3 = k$ are always real numbers.

5

4. (a) The point A(2, 2) lies on the parabola
$y = x^2 + px + q$.
Find a relationship between p and q.
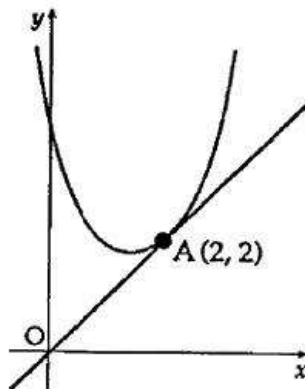
(1)

(b) The tangent to the parabola at A is the line $y = x$. Find the value of p. Hence find the equation of the parabola.

(6)

(c) Using your answers for p and q, find the value of the discriminant of $x^2 + px + q = 0$. What feature of the above sketch is confirmed by this value ?

(2)

5. (a) $f(x) = 2x + 1$, $g(x) = x^2 + k$, where k is a constant.
  (i) Find $g(f(x))$.

(2)
  (ii) Find $f(g(x))$.

(2)

(b) (i) Show that the equation $g(f(x)) - f(g(x)) = 0$ simplifies to
$2x^2 + 4x - k = 0$.

(2)
  (ii) Determine the nature of the roots of this equation when $k = 6$.

(2)
  (iii) Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots.

(3)

6. Diagram 1 shows a rectangular plate of transparent plastic moulded into a parabolic shape and pegged to the ground to form a cover for growing plants. Triangular metal frames are placed over the cover to support it and prevent it blowing away in the wind.

Diagram 2 shows an end view of the cover and the triangular frame related to the origin O and axes Ox and Oy. (All dimensions are given in centimetres.)
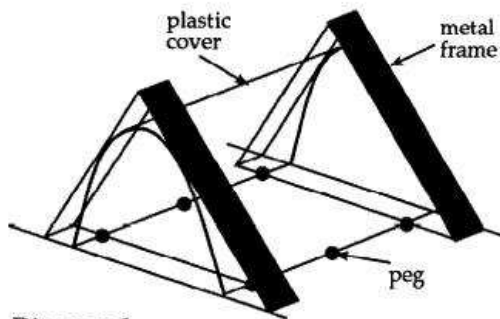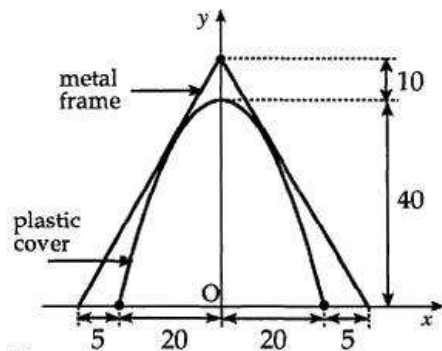

Diagram 1

Diagram 2

(a) Show that the equation of the parabolic end is $y = 40 - \dfrac{x^2}{100}, -20 \le x \le 20$. **(4)**

(b) Show that the triangular frame touches the cover without disturbing the parabolic shape. **(7)**

7.

(a) Write the equation $\cos 2\theta + 8\cos\theta + 9 = 0$ in terms of $\cos\theta$ and show that, for $\cos\theta$, it has equal roots. **3**

(b) Show that there are no real roots for θ. **1**

8. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle? **5**

9. For what value of k does the equation $x^2 - 5x + (k+6) = 0$ have equal roots? **3**

10. Calculate the least positive integer value of k so that the graph of $y = kx^2 - 8x + k$ does not cut or touch the x-axis. **4**
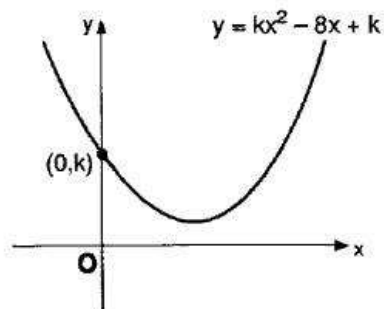
11. Find the values of k for which the equation $2x^2 + 4x + k = 0$ has real roots. **2**

12. The roots of the equation $(x - 1)(x + k) = -4$ are equal.

Find the values of k. **5**

13. For what value of a does the equation $ax^2 + 20x + 40 = 0$ have equal roots? **2**

14. Show that the equation $(1 - 2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k. **5**

15. Find the possible values of k for which the line $x - y = k$ is a tangent to the circle $x^2 + y^2 = 18$. **5**

[END OF QUESTIONS]